Categorical centers and Yetter Drinfel‘d-modules as 2-categorical (bi)lax structures

(joint work with Sebastian Halbig)

Center categories of monoidal categories $\mathcal{C}$ and of bimodule categories $\mathcal{M}$ are very well known and studied in the literature. We consider the (weak) center category $\mathcal{Z}(F, \mathcal{M}, G)$ of a $\mathcal{C}$-$\mathcal{D}$-bimodule category $\mathcal{M}$ twisted by two lax monoidal functors $F : \mathcal{E} \to \mathcal{D}$ and $G : \mathcal{E} \to \mathcal{C}$, for another monoidal category $\mathcal{E}$. (The weakness corresponds to dealing with half-braidings, while with strongness we allude to (invertible) braidings.)

We show how the 2-categorical viewpoint provides a deeper insight on such center categories. We formulate a bicategory of weak left (resp. right) centers categories and show how a full sub-bicategory of both of them recovers the bicategory $TF(\mathcal{C}, \mathcal{D})$ from [4, Section 3]. Moreover, we prove a more general result in bicategories by which the rigidity of $TF(\mathcal{C}, \mathcal{D})$ is recovered.

On the other hand, we introduce a 2-category $\mathrm{Bilax}(\mathcal{K}, \mathcal{K}')$ of bilax functors (among 2-categories $\mathcal{K}$ and $\mathcal{K}'$). Its 0-cells are a 2-categorification of bilax functors of [1] and of bimonoidal functors of [3]. We show how bilax functors generalize the notions of bialgebras in braided monoidal categories, *bimonads* in 2-categories (with respect to Yang-Baxter operators, YBO's), and preserve bimonads (w.r.t. YBO's), *module comonads* and *comodule monads*, and *relative bimonad modules*. Moreover, the component functors of a bilax functor on hom-categories factor through the category of *Hopf bimodules* (w.r.t. YBO's). (The 2-categorical notions in italic letters are introduced in our work.)

We finally show that there is a 2-category isomorphism $\mathrm{Bilax}_c(1, \mathcal{K}) \cong \mathrm{Bimnd}(\mathcal{K})$ and a faithful 2-functor $\mathrm{Bimnd}(\mathcal{K}) \hookrightarrow \mathrm{Dist}(\mathcal{K})$. Here $\mathrm{Bimnd}(\mathcal{K})$ is the 2-category of bimonads from [2] and $\mathrm{Dist}(\mathcal{K})$ is the 2-category of mixed distributive laws of [5].

References

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